[2] Eliashberg, V. M., Dissertation, competition for degree, Candidate of Technical Science, Leningrad Technical Institute m. Lensovet, Leningrad (1960).

[3] Burovoi, I. A. and Eliashberg, V. M., *Avtomatizatsiia v Proizvodstve Kislot i Udobrenii* (*Automatization in the Acid and Fertilizer Industries*), GKhI (State Chemical Press, 1960).

[4] "Review of the literature on the problem of mathematical simulation," Avtomatika i telemekhan. (Automat. and Remote Control), no. 11, 12 (1959).

[5] Brodskii, A. M., Kalinenko, R. A., and Rozental', K. P., Zh. Fiz. Chem. (J. Phys. Chem.) **XI,** no. 1, 29 (1960).

Reviewer's Comment

This paper contributes to the literature a procedure of mathematical simulation of dynamics in the fluidized solids equipment by means of an analog computer. Several schemes of mathematical simulation are given for some simple cases.

Provided that the mass transfer and chemical reaction follow the simple models assumed in this paper, the proposed mathematical simulation is acceptable. However, there are a number of other factors that can control the overall transfer or reaction rates in the fluidized beds. For instance, discussion of the formation of bubbles, dispersion of fine solid particles in the bubbles, contact efficiency between the gas stream and solid particles, and back mixing, etc., is lacking in this paper. Therefore, readers should take note of the restricted assumptions, employed here for the establishment of the fundamental equations.

Only the diagrams of simulation of the proposed fundamental equations are presented in this paper, and the reviewer thought it would be more valuable if it had included several results of numerical calculation, preferably as the dimensionless charts. Then the reader could compare his experience using the fluidized solids equipment with the mathematical treatments proposed in this paper or could evaluate the adequacy of the fundamental assumptions.

—Sakae Yagi
Professor of Chemical Engineering
University of Tokyo, Japan

MAY 1963 AIAA JOURNAL VOL. 1, NO. 5

Determination of the Probability of Loss of Stability by a Shell

V. M. Goncharenko

The calculation of stability of a shell is the establishment of a critical value λ_* of the loading parameter λ such that for $\lambda < \lambda_*$ the shell is guaranteed to be in a state considered acceptable for operation. According to the theory of stability of shells (e.g., Ref. 1), a recommended value for λ_* is the upper critical loading λ_+, calculated for shells that are ideal in the sense that imperfections of form, material, and the like may be ignored in its computation. As a basis for this we may reason that the unperturbed state is stable in the sense of Liapunov only for $\lambda < \lambda_+$. However, comparison of theoretical data with the results of experiments[2] shows that the actual λ_* lies significantly below the value computed from classical theory. At present this fact is explained[3–5] by the observation that the stability of shells is significantly influenced by the distribution of characteristics of the shell on the one hand and by perturbations of the loading on the other.

In the present note we wish to indicate that the influence of the forementioned groups of factors has a different character.

Let the characteristics of the shell and the nature of its support be described by a system of parameters $a_j (j = 1,2, \ldots, m)$ which are random variables with joint probability density function $\varphi(a_j)$, known from experiment. Let the one-dimensional probability density function of the loading λ be denoted by $\psi(\lambda)$. In the general case $\psi(\lambda)$ depends on the time; that is, λ is a random process. Let us assume, following Refs. 3 and 5, that the spectral density of $\lambda(t)$ is different from zero only in the frequency range $\omega \ll \omega_0$, where ω_0 is the lowest natural frequency of oscillation of the shell. At $t = t_0$ we put the shell in an unperturbed equilibrium state with zero initial velocity and then observe its evolution under the influence of a random loading $\lambda(t)$. If other factors of a dynamical character are absent, then it follows from our assumptions that this evolution will be through a sequence of equilibrium states, and, from the definition of λ_+, that in each separate case loss of stability occurs at $\lambda = \lambda_+(a_j)$. One may conclude that the distribution of geometric and physical properties of the shell, the method of support, and the loadings for its quasi-static motion reduce simply to the corresponding distribution of the upper critical loading. The possibility of loss of stability for $\lambda_-(a_j) < \lambda < \lambda_+(a_j)$, where λ_- is the lower critical loading, depends on dynamical perturbing factors having energy sufficient to overcome the potential barrier. Thus the method of Bolotin[3,5] (in that part where the concept "quasi-static" is used) may be considered a means of studying the distribution of the upper critical loading.

From the preceding reasoning there follows a method of determining the probability of loss of stability, somewhat different from that described in Refs. 3 and 5. Under the assumption of a quasi-static loading, we may write

$$P(*|a_j,\lambda) = \sigma_0[\lambda - \lambda_+(a_j)] \tag{1}$$

Translated from Izvestiia Akademii Nauk SSSR, Otdel. Tech. Nauk, Mekhanika i Mashinostroenie (Bulletin of the Academy of Sciences of the USSR, Div. Tech. Sci., Mechanics and Machine Construction), no. 1, 159–160 (1962). Translated by Warren S. Loud, University of Minnesota.

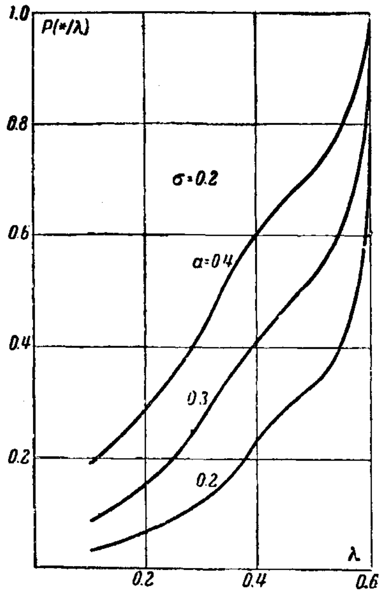

Fig. 1

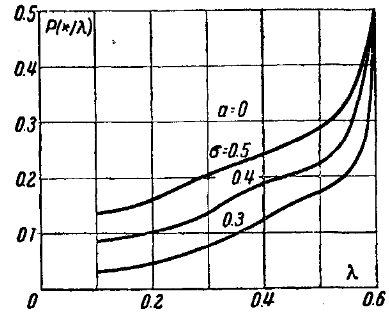

Fig. 2

Here $P(*|a_j,\lambda)$ is the conditional probability of loss of stability for fixed a_j,λ; σ_0 is the function of Heaviside. According to the theorem of total probability

$$P(*) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} P(*|a_j,\lambda)\varphi(a_j)\psi(\lambda)da_j d\lambda = \int_{\lambda>\lambda_+} \cdots \int_{(a_j)} \varphi(a_j)\psi(\lambda)da_j\, d\lambda \quad (2)$$

Formula (2) in principle takes into account distribution of the parameters of the shell and the "slow" fluctuation of the loading. The upper critical loading $\lambda_+(a_j)$ in this case is to be determined with regard to the deviation of a_j, and also boundary conditions, minimization with respect to wave numbers, etc.

For example, let us consider a circular cylindrical shell of length L and thickness h. Let its compression modulus and Poisson coefficient be E and ν. Let us assume that as a result of inaccuracies of manufacture the radius of the shell is random and equal to $R + f_0h$, where f_0 is a random variable with probability density function $\varphi(f_0)$. In mounting the shell is fixed to ribs with radius R, and thus receives an initial deflection equal at the edge to $-f_0h$, and also a corresponding initial strain. The shell is then loaded with axial stresses

$$T = (Eh^2/R)\lambda \quad (3)$$

When the initial deflection is directed toward the center of curvature ($f_0 > 0$), the upper critical value with corresponding minimization with respect to half-wave number, valid for shells which are not too short, can be found in the monograph of Ref. 2. It is determined from the following equation:

$$0.563(1 - \nu^2)^{-1/2} - 0.842\lambda = (f_0 + \nu\lambda_+)(1 - 1.81\theta)\sqrt{1 - \lambda_+\sqrt{3(1 - \nu^2)}} \quad (4)$$

Here

$$\theta = \frac{\sqrt{Rh}}{L\sqrt{2}(1 - \nu^2)^{1/4}} \quad (5)$$

The upper and lower critical values of λ, calculated for an "ideal" shell ($f_0 = 0$), are 0.6 and 0.18. There is reason to suppose that for initial deflections directed away from the center of curvature ($f_0 < 0$), $\lambda_+ > 0.6$. It is not difficult to show that in the range of values of λ_+ which is of interest, the dependence of λ_+ on f_0, and of f_0 on λ_+ is single-valued and monotonic. Therefore, for $\lambda < 0.6$ we have

$$P(*|\lambda) = \int_m^{\infty} \varphi(f_0)\, df_0 \quad (6)$$

where m is found by means of

$$m = \frac{0.563(1 - \nu^2)^{-1/2} - 0.842\lambda}{(1 - 1.81\,\theta)\sqrt{1 - \lambda_+\sqrt{3(1 - \nu^2)}}} - \nu\lambda \quad (7)$$

If it is assumed that f_0 is normally distributed with mean value a and variance σ^2, formula (6) takes the form

$$P(*|\lambda) = \frac{1}{2}\left[1 - \Phi\left(\frac{m - a}{\sigma}\right)\right] \quad (8)$$

where Φ is the probability integral.

In Fig. 1 is shown the dependence of $P(*|\lambda)$ on λ for $\nu = 0.3$, $\theta = 0.0464$, $\sigma = 0.2$, for various values of a. In Fig. 2 the same dependence is shown for $a = 0$ and various values of σ. It is necessary to note the presence of points of inflection on the curves indicated in the interval $0.18 < \lambda < 0.6$. From the relation

$$P(*|\lambda) = \int_{-\infty}^{\lambda} f(\lambda_*)\, d\lambda_* \quad (9)$$

where $f(\lambda_*)$ is the probability density function of the critical loading, one may draw conclusions about the existence of extrema of the distribution $f(\lambda_*)$ at the indicated points. A simple analysis shows that $f(\lambda_*)$ has two maxima: one is found close to $\lambda_+ = 0.6$ and the other displaced on the side of λ. (The existence of similar maxima was first discovered by Makarov.[6])

—*Received July 2, 1961*

References

[1] Timoshenko, S. P., *Stability of Elastic Systems* (Gostekhizdat, 1946).

[2] Mushtari, H. M. and Galimov, K. Z., *Nonlinear Theory of Elastic Shells* (Tatknigizdat, 1957).

[3] Bolotin, V. V., "Statistical methods in nonlinear theory of elastic shells," Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk (Bull. Acad. Sci. USSR, Div. Tech. Sci.), no. 3 (1958).

[4] Vorovich, I. I., "Statistical methods in the theory of stability of shells," Prikl. Mat. i Mekhan. (Appl. Math. and Mech.) **23,** no. 5 (1959).

[5] Bolotin, V. V., *Statistical Methods in Construction Mechanics* (Construction Industry Press, 1961).

[6] Makarov, B. P., "Application of statistical methods to the analysis of experimental data in the stability of shells," Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Mekhan. i Mashinostr. (Bull. Acad. Sci. USSR, Div. Tech. Sci., Mech. and Machine. Constr.)